

# Measurement and its Relationship to Mathematics: Complexity within Young Children's Beliefs

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A qualitative investigation of young children's beliefs included the gaining of insights into beliefs about measurement and its relationship to mathematics, the focus of this paper. Through the use of a range of procedures in one-to-one interviews, conducted with eight children over a period of five months, beliefs were found to be complex and idiosyncratic. Children may see measurement differently from how adults see it, and may hold different meanings for measurement terminology. This has implications for teachers of mathematics in the primary school.

Based on a doctoral study of young children's beliefs about the nature of mathematics, the nature of learning, and helping factors for the learning of mathematics, this paper focuses on beliefs related to the nature of mathematics. The research question addressed is "What beliefs do children hold about the nature of mathematics?" Previously the author reported on one aspect of one child's beliefs about the nature of mathematics (McDonough, 1996). The present paper takes a different approach, focusing on a broader theme, beliefs about measurement and its relationship to mathematics, and drawing on findings gained from the indepth study of eight children's beliefs.

## Why Investigate Children's Beliefs?

It is important that educators are aware of children's beliefs as they may influence children's interpretations of learning situations and may impact upon learning of mathematics. Beliefs may have more influence "than knowledge in determining how individuals organize and define tasks and problems and are stronger predictors of behaviour" (Pajares, 1992, p. 311). Perceptions of what mathematics is may influence approaches to the solving of problems in mathematics (Frank, 1988), may influence the nature of children's participation in meaningful mathematics learning (Franke & Carey, 1997), may be related to success in mathematics (Crawford, Gordon, Nicholas, & Prosser, 1993), may impact upon conceptions of specific topics in mathematics, and may affect attitudes, performance, confidence, perceived usefulness of mathematics, and choice of courses or careers (Kouba & McDonald, 1987). Similarly, the learner's everyday knowledge of what is mathematics can influence the learner's interpretation of what is taught (Lindenskov, 1993) and consequently influence what is learned.

According to Ernest (1996), little systematic research has been undertaken on images of mathematics, with limited investigation of the views of students. The research reported here has undertaken such investigation and provides insights not readily available in day-to-day workings of the classroom. As an aside, the procedures developed within the study are potentially of use to other researchers and to teachers wishing to seek insights into individual children's beliefs.

Beliefs are generally stable (McLeod, 1992; Pajares, 1992), and therefore can have a consistent influence on learners, but there is the possibility also of change depending on the degree of non-centrality of beliefs (Rokeach, 1968). It is possible therefore that input from significant others, such as spoken views of teachers, impacts upon children's construction of beliefs. Although some studies, such as that reported in this paper, reveal differences in beliefs held by students taught the same program by the same teacher (e.g., McDonough, 2002; Rodd, 1993), it appears that curricula can impact upon the development of students' beliefs about mathematical activity (Franke & Carey, 1997). Other experiences or elements in students' lives may be of influence also. Having insights into the complexity and individuality of children's beliefs such as provided in the present study, or knowing the beliefs of individual learners in one's own mathematics classroom, can inform teachers for their communication with students and assist in making decisions for teaching. Although teachers may not be able to change the curriculum, they can cater for students in a more informed manner. In addition, it is of value to investigate beliefs about measurement as this is a key strand of the mathematics curriculum (e.g., Board of Studies, 2000) and, as such, potentially receives some emphasis in the primary mathematics classroom.

### Theoretical Orientation of the Research

The study's focus on beliefs was based on a social constructivist view of learning, as expressed by Ernest (1991), where there is a "respect for each individual's rights, feelings and sense-making ... [and] children and other persons are seen as active and enquiring makers of meaning" (p. 198). Knowledge is considered to be individually constructed and socially determined. The research extended this view, making the assumption that beliefs, as well as understandings, are individually constructed (e.g., Yackel & Cobb, 1996), and that these, in turn, may affect learning.

The study was qualitative in purpose, that is, it had an "interest in human meaning in social life and its elucidation and exposition by the researcher" (Erickson, 1986, p. 119). Qualitative research, in contrast to the "traditional" or "scientific" paradigm, ... assumes there are multiple realities—that the world is not an objective thing out there but a function of personal interaction and perception" (Merriam, 1988, p. 17). Taking into account children's varying experiences and individual constructions, the research was underpinned by the expectation that beliefs differ from child to child.

### Children's Beliefs about the Nature of Mathematics

Much research suggests that primary school children tend to have narrow conceptions of mathematics, focused mainly on number, including counting and the operations (e.g., Cotton, 1993; Frank, 1988; Garafolo, 1989; McDonald & Kouba, 1986; Spangler, 1992; Stodolsky et al., 1991). Students believe that mathematical activity at school includes memorising formulas and, in a rote fashion, carrying out computations and procedures (Garafolo, 1989; Schoenfeld, 1992; Spangler, 1992) that are "divorced from real life, from discovery and from problem solving" (Schoenfeld, 1987, p. 197). The emphasis on mathematics as a rule-governed activity appears present within students both at the primary and secondary levels (Cobb, 1985).

In contrast, different views of what it means to do mathematics were found in a study of the views of 36 first-grade children from six Cognitively Guided Instruction (CGI) classrooms (Franke & Carey, 1997). In these classrooms "students had opportunities to

consistently engage in problem solving, discuss their solution strategies and build on their own informal strategies for solving problems" (Franke & Carey, 1997, p. 10). In discussing what it means to do mathematics, the children referred to "problem solving, use of manipulatives, talking about mathematics, and solving problems in a variety of ways" (p. 14). Beliefs differing from the traditional view of mathematics were found also among successful secondary mathematics students who saw mathematics as something in which one can be creative and discover things (Wood & Smith, 1993).

The reported focus on number within children's beliefs suggests limited recognition and/or suggestion by primary children of measurement as mathematical activity. Indeed, in one study of the beliefs of 1202 primary school children, teachers were surprised by their children's narrow perception of what is mathematics, including little reference to measurement (McDonald & Kouba, 1986).

The study reported in this paper gives insights into young children's beliefs that extend our knowledge of children's perspectives. This occurred through an indepth investigation.

## Research Methods

The research took the form of individual case studies of eight children of eight to nine years of age from two schools in suburban Melbourne. The children were chosen by their teachers according to gender and achievement guidelines provided by the researcher. Table 1 gives summary details of the research participants, according to the pseudonyms given to the children.

Table 1

*Summary of Details of Child Participants in the Study.*

Child	Anna	Ben	Cara	David	Emily	Filip	Gina	Harry
School	S	S	S	S	I	I	I	I
Gender	female	male	female	male	female	male	female	male
Achievement (teacher rated)	high	high	low	low	high	high	low	low

As the main form of data collection, the children were each interviewed by the researcher for about 30 minutes on ten occasions over a five-month period using 30 semi-structured, creative interviewing procedures (Patton, 1990, p. 340) that were developed or adapted for the study. These included drawing, discussing scenarios presented through photographs, video snippets and other children's drawings, ordering of descriptors, writing, and responding to questionnaires presented verbally.

Of the 30 procedures used in the study, 14 were targetted specifically at investigating beliefs about the nature of mathematics. For example, in Task 5.2, adapted from Zevenbergen and Crowe (1992), the children were presented with 22 photographs representing school mathematics contexts, non-school mathematics contexts, and non-mathematical or ambiguous contexts (e.g., group posing in front of a mountain scene). Children were asked to describe what they believed was happening in each photograph and then asked whether they believed there was any maths in what the person(s) was doing.

Children were also given opportunities to proffer mathematical situations of their own choice. For example, in Task 4.2, children were asked to draw a picture of someone using

or doing maths and to describe their picture. Discussion focused on what the person was doing, and in what way the activity was mathematical.

Analysis of each child's interview responses was undertaken through "criss-crossed" reflection (Stake, 1994) through which themes (van Manen, 1990) were drawn. Data on beliefs about the nature of mathematics came from a range of procedures increasing the credibility and authenticity of the findings, important elements of qualitative research (LeCompte & Goetz, 1982). For example, the number of procedures giving insights into beliefs about mathematics, that were drawn upon in the analysis and write-up in the thesis, ranged from 12 to 18 per child. Children's responses were not judged for a match to any predetermined categories and the researcher sought to take a stance of neutrality to the phenomena under study, again increasing the credibility of the study. Further themes, such as those related to beliefs about measurement as discussed below, were then identified across the data from the eight children.

## Results and Discussion

In presentation and discussion of results the term *maths*, the common abbreviation of *mathematics* (Wilkes & Krebs, 1982), is used to mirror the terminology use in the interviews. Not all children held the same meaning for the terms *maths* and *mathematics* but the former was a term familiar to all eight children.

Themes that emerged from the children's responses regarding their perceptions of measurement as maths included the following:

- children vary in their beliefs as to whether aspects of formal measurement are maths;
- children vary in their beliefs as to whether informal measurement is maths;
- when speaking of measuring, children are not necessarily speaking of maths;
- children may hold idiosyncratic meanings for commonly used terms related to measurement.

Each of the above points is considered in the following discussion.

At the time of the study, the state mathematics curriculum document recommended that Grade 3 children be introduced to some formal units of measure preceded by informal measurement activities (Board of Studies, 1995). During the year of the data collection, measurement was included in the mathematics programs of the two classes from which the children came. However, Ms S, the teacher of Anna, Ben, Cara and David reported that the children had been taught less measurement than she would have liked. Use of both formal and informal units was reported to have been included in the measurement lessons. When Ms I, the teacher of Emily, Filip, Gina and Harry, was asked how much emphasis she had given to different aspects of the mathematics program, choosing from *a lot*, *some*, *not much*, or *not at all*, she reported that she had given *some* emphasis to measurement.

Data indicate that for the children referred to as Anna, Cara, David, Emily and Filip, measuring was linked mostly to the use of formal units. For example, when discussing a photograph of a child measuring another child with string, Cara referred also to measurement of length in the school situation, but indicated that she used a tape measure. Formal units of measure for mathematical activity were mentioned or implied at times when speaking of her father making cakes such as in her second interview when Cara stated, "He gets a jug that goes up to 150 and he puts the cream in". She spoke also of him using a ruler to measure a circle for a wedding cake. Formal units were referred to specifically when Cara described a photograph presented to her that she interpreted as a girl

reading a packet and baking a cake. She stated: "she's seeing on the back here to see if it's 1 mil (mL) of water or 2 mils (mL), and there she's putting the water in". Cara also referred to mass formal measurement situations as mathematical.

David provided a further example of the identification of measurement, and most specifically formal measurement, as mathematical. David included measure and measurement within his maths word wheel response (a type of brainstorm in response to the question "What is maths?") and stated in another interview that "in maths you add up things and measure them". Measuring objects with a ruler or a measuring tape was identified as mathematical activity. David said that in one situation there was no maths "because you use rulers and other things in your own pencil case [for maths]". Situations posed that involved informal measurement through direct comparison were judged as not mathematical; for one the justification was that "you usually use a ruler" for maths. David's responses suggested that rulers and tape measures, as well as a range of scales, were tools that for David indicated mathematical activity. The tools involved the use of formal units of measure.

However, there were instances where the presence of such measurement tools and/or formal units did not guarantee that situations would be considered mathematical. For example, two mass situations, presented in photographic form and involving the use of formal units, were not considered by Emily as mathematical activity. A photograph of a woman using a graduated circular scale to weigh fruit at a supermarket was described as "The lady is weighing something and seeing how many kilos or grams it is". When asked whether there was any maths in what she was doing Emily seemed undecided but tended towards the negative: "um, um, mmm, mmm no", explaining that it was not maths because "she's just weighing something". Similarly, another photograph was described as a "man caught a big fish, holding it up and weighing how many kilos". Emily thought that this also was not maths. Emily's statements suggest that she may not have considered weighing to constitute mathematical activity. However, it is possible that she might have been looking for the presence of counting or an operation with numbers. She had shown an appreciation of the measurement of length using formal units as mathematical activity, but seemingly mainly because of the application of number through counting or addition. The numbers on the supermarket scale were visible in the photograph, their purpose was understood by Emily, but it appears Emily did not perceive their use on this occasion as mathematical.

These examples show that differences in children's beliefs about formal measurement as maths relate to aspects including measurement attributes, units of measure, and measurement terminology.

Links between informal measurement and maths were complex. It is noted that the informal measurement situations presented to the children included direct comparison and indirect comparison through use of a third object. Measurement using informal units was not included. In retrospect, inclusion of such situations may have given further insights. However, children did have opportunities to refer to such tasks in the open questions requesting examples of mathematical activity but none chose to do so. The measurement-by-comparison tasks do give interesting results.

Situations that were classified by the researcher as involving informal measurement were considered by two children, Gina and Ben, as measurement and as maths. These included an item presented verbally regarding a child laying a pencil next to a book to see which was longer, and a photograph of a child measuring the height of another using a piece of string. Gina and Ben also saw formal measurement as maths.

Anna held mixed views about the informal measurement activities posed to her: using string as a tool for comparing lengths was considered maths, but informal measurement through direct comparison was seen as guessing and not maths.

Cara referred to informal measurement-by-comparison situations as *maths* but not as *measurement*. It appears that *measurement*, as Cara saw it, was a subset of maths that included formal measures and thus numbers. Informal measurement-by-comparison situations presented by the researcher appeared to belong, for Cara, to a different part of *maths*, that did not include measurement, suggesting Cara believed measurement required numbers but maths did not.

Emily at times made an association between measurement and maths, but mainly when numbers, counting or adding were involved. Emily called informal measurement situations *measuring* but said they were not *maths*, the latter seemingly due to her need to identify number in a mathematical situation.

As discussed above, although Emily used terms such as measure, kilograms, grams, and weighing in describing situations, this did not ensure that she saw the activities as mathematical; she did not make an automatic association between measurement terms and maths. Harry also used the word measuring along with other measurement terms, including names of formal units such as “kilos”, and measurement tools such as rulers, but did not indicate that he associated these with maths.

Responses such as those from Emily and Harry, where measurement terms were used but not associated with mathematics indicate that the understanding of what might be considered familiar language of mathematics is not common to all, although the same words, such as measuring or weighing, may be uttered. The results suggest that modelling and discussion of mathematical language is an important role for teachers of young children, and a role of which teachers can be conscious.

There are questions also about children's understandings of the relationship between informal and formal measurement. All of the children except Harry identified formal measurement situations as maths but six of the children either did not recognise informal measurement-by-comparison situations as maths or had mixed beliefs about the different situations posed by the researcher. As stated above, none proffered a situation with informal units as mathematical activity. This suggests that links recognised by adults should not be assumed for young children. A related point emerging from the study concerns the importance of informal measurement and the emphasis it should be given in the mathematics curriculum, a question of debate in the mathematics education community (e.g., Ainley, 1990). As this issue is not related directly to the research questions of the current study it will not be considered here but the responses from the children suggest investigation of the value of informal measurement in the mathematics classroom is worthwhile.

## Conclusion

The teaching of measurement in the primary school involves the development of a range of concepts and skills. It may involve the use of problem solving processes if measurement problems are posed by the children or teacher. The present study indicates that although a teacher may include measurement in the mathematics program, children may not view this as mathematical activity. Children come to the learning situation with their own beliefs and, even for an aspect such as measurement, these may differ from child to child and from those of the teacher.

Two of the eight children in the study, Cara and Ben, experienced measurement in real life situations involving family members, and called on these experiences of food preparation and building when discussing maths. They did not see maths or measurement as only school-related activity. Cara also held a strong association between measurement and estimating and included the latter as mathematical activity. Indeed, these were the two activities she seemed to associate most with maths. The other children identified some situations posed by the researcher as measurement but proffered fewer instances than Cara or Ben.

These findings suggest the key role that families, particularly parents, can play in the development of children's beliefs. The research supports the message that it is desirable for parents to feel comfortable with their own use of mathematics and involve children in real, rather than contrived, mathematical activities in their daily lives. Schools might consider working with parents to examine appropriate mathematical activities to undertake with children in non-school environments in a relaxed and non-contrived manner.

The difference in beliefs regarding the relationship of informal measurement to maths, and of different measurement concepts to maths suggests value in teachers posing a range of problems to children and reflecting, with the children, on the solving of these as mathematical activity. The research findings suggest also that measurement attributes, units of measure, and measurement terminology might be considered when focusing children's reflections.

This research shows that young children's beliefs about measurement are idiosyncratic and that associations between measurement, its terminology, and mathematics, may be different from common understandings. Teachers should not assume the beliefs that children hold. As beliefs may impact upon children's reactions to, or interpretations of, what is stated, performed or produced in a mathematics learning situation and may affect many aspects of their learning, it is worthwhile to gain insights into children's beliefs. Results from the current study suggest that the use of more than one procedure on multiple occasions can show detail and complexity within young children's beliefs that might not otherwise be available and that can inform teaching and interactions with children.

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